PITAGORAS
ACADEMIA

TRIGONOMETRÍA

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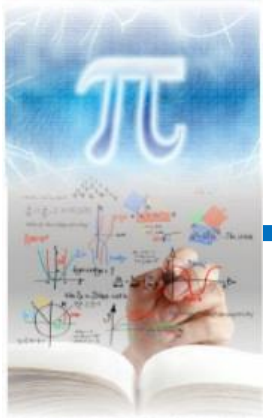
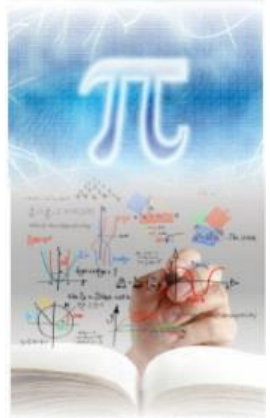
IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO





IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

IDENTIDADES FUNDAMENTALES:

1. IDENTIDADES POR COCIENTE :

$$\text{Tanx} = \frac{\text{Senx}}{\text{Cosx}} \qquad \forall\, x \in \mathbb{R} - (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$$

$$\text{Cotx} = \frac{\text{Cosx}}{\text{Senx}} \qquad \forall\, x \in \mathbb{R} - n\pi; n \in \mathbb{Z}$$

2. IDENTIDADES RECÍPROCAS:

$$\text{Senx}.\,\text{Cscx} = 1 \qquad \forall\, x \in \mathbb{R} - n\pi\,; n \in \mathbb{Z}$$

$$\text{Cosx}.\,\text{Secx} = 1 \qquad \forall\, x \in \mathbb{R} - (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$$

$$\text{Tanx}.\,\text{Cotx} = 1 \qquad \forall\, x \in \mathbb{R} - \frac{n\pi}{2}; n \in \mathbb{Z}$$

3. IDENTIDADES PITAGÓRICAS :

$$\text{Sen}^2x + \text{Cos}^2x = 1 \qquad \forall\, x \in \mathbb{R}$$

$$1 + \text{Tan}^2x = \text{Sec}^2x \qquad \forall x \in \mathbb{R} - (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$$

$$1 + \text{Cot}^2x = \text{Csc}^2x \qquad \forall x \in \mathbb{R} - n\pi; n \in \mathbb{Z}$$

4. PROPIEDAD:

Si: $a\text{Senx} + b\text{Cosx} = c$; a, b y c $\in \mathbb{R}$

además: $a^2 + b^2 = c^2$

entonces:

$$\text{Senx} = \frac{a}{c} \quad \wedge \quad \text{Cosx} = \frac{b}{c}$$

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

IDENTIDADES AUXILIARES:

a) $(\text{Senx} \pm \text{Cosx})^2 = 1 \pm 2\text{SenxCosx}$

b) $\text{Sen}^4\text{x} + \text{Cos}^4\text{x} = 1 - 2\text{Sen}^2\text{xCos}^2\text{x}$

c) $\text{Sen}^6\text{x} + \text{Cos}^6\text{x} = 1 - 3\text{Sen}^2\text{xCos}^2\text{x}$

d) $\text{Sen}^8\text{x} + \text{Cos}^8\text{x} = 1 - 4\text{Sen}^2\text{xCos}^2\text{x} + 2\text{Sen}^4\text{xCos}^4\text{x}$

e) $\text{Tanx} + \text{Cotx} = \text{Secx}.\,\text{Cscx}$

f) $\text{Sec}^2\text{x} + \text{Csc}^2\text{x} = \text{Sec}^2\text{x}.\,\text{Csc}^2\text{x}$

g) $\text{Tan}^2\text{x} - \text{Sen}^2\text{x} = \text{Tan}^2\text{x}.\,\text{Sen}^2\text{x}$

h) $\text{Cot}^2\text{x} - \text{Cos}^2\text{x} = \text{Cot}^2\text{x}.\,\text{Cos}^2\text{x}$

i) $(1 \pm \text{Senx} \pm \text{Cosx})^2 = 2(1 \pm \text{Senx})(1 \pm \text{Cosx})$

j) $\dfrac{\text{Senx}}{1 \mp \text{Cosx}} = \dfrac{1 \pm \text{Cosx}}{\text{Senx}}$

k) $\dfrac{\text{Cosx}}{1 \pm \text{Senx}} = \dfrac{1 \mp \text{Senx}}{\text{Cosx}}$

l) $\text{Secx} + \text{Tanx} = \text{N} \longleftrightarrow \text{Secx} - \text{Tanx} = \dfrac{1}{\text{N}}$

m) $\text{Cscx} - \text{Cotx} = \text{M} \longleftrightarrow \text{Cscx} + \text{Cotx} = \dfrac{1}{\text{M}}$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

1. Si: $\sqrt{5}\text{Cosx} + 2 = 3\text{Tan}^2\text{x}$ calcule el valor de: $9\text{Tan}^6\text{x} - 3\text{Tan}^4\text{x} - 8\text{Tan}^2\text{x}$

Resolución:

$$\sqrt{5}\text{Cosx} + 2 = 3\text{Tan}^2\text{x}$$

$$\sqrt{5}\underbrace{\text{Cosx} \times \text{Secx}}_{1} = (3\text{Tan}^2\text{x} - 2) \times \text{Secx}$$

$$\sqrt{5} = (3\text{Tan}^2\text{x} - 2)\text{Secx}$$

AL^2: $5 = (3\text{Tan}^2\text{x} - 2)^2\text{Sec}^2\text{x}$

$$5 = (9\text{Tan}^4\text{x} - 12\text{Tan}^2\text{x} + 4)(1 + \text{Tan}^2\text{x})$$

$$5 = 9\text{Tan}^4\text{x} - 12\text{Tan}^2\text{x} + 4 + 9\text{Tan}^6\text{x} - 12\text{Tan}^4\text{x} + 4\text{Tan}^2\text{x}$$

$$5 = 9\text{Tan}^6\text{x} - 3\text{Tan}^4\text{x} - 8\text{Tan}^2\text{x} + 4$$

$$\therefore \mathbf{9\text{Tan}^6\text{x} - 3\text{Tan}^4\text{x} - 8\text{Tan}^2\text{x} = 1}$$

CLAVE: A

2. Dada la identidad trigonométrica, para un ángulo agudo: $\text{Cscx}\sqrt{\text{Cotx}} + \text{Secx}\sqrt{\text{Tanx}} = (\text{Tanx} + \text{Cotx})^{1/k}$

Entonces el valor de k es:

Resolución:

$$\text{Cscx}\sqrt{\text{Cotx}} + \text{Secx}\sqrt{\text{Tanx}} = (\text{Tanx} + \text{Cotx})^{1/k}$$

$$\frac{1}{\text{Senx}}\frac{\sqrt{\text{Cosx}}}{\sqrt{\text{Senx}}} + \frac{1}{\text{Cosx}}\frac{\sqrt{\text{Senx}}}{\sqrt{\text{Cosx}}} = (\text{SecxCscx})^{1/k}$$

$$\frac{\overbrace{\text{Cos}^2\text{x} + \text{Sen}^2\text{x}}^{1}}{\text{SenxCosx}\sqrt{\text{SenxCosx}}} = \left(\frac{1}{\text{SenxCosx}}\right)^{1/k}$$

$$\frac{1}{(\text{SenxCosx})(\text{SenxCosx})^{1/2}} = \left(\frac{1}{\text{SenxCosx}}\right)^{1/k}$$

$$\left(\frac{1}{\text{SenxCosx}}\right)^{3/2} = \left(\frac{1}{\text{SenxCosx}}\right)^{1/k}$$

$$\frac{3}{2} = \frac{1}{k}$$

$$\therefore \mathbf{k = \frac{2}{3}}$$

CLAVE: C

3. Si la expresión: $P = a(2Sen^4x + Cos^4x + Cos^2x) - b(Sen^6x + Cos^6x)$; es independiente de x, calcule a/b

Resolución:

$$P = a(2Sen^4x + Cos^4x + Cos^2x) - b(Sen^6x + Cos^6x)$$

$$P = a(\underbrace{Sen^4x + Cos^4x}_{1 - 2Sen^2xCos^2x} + \underbrace{Sen^4x}_{Sen^2xSen^2x} + 1 - Sen^2x) - b(\underbrace{Sen^6x + Cos^6x}_{1 - 3Sen^2xCos^2x})$$

$$P = a[1 - 2Sen^2xCos^2x + 1 - Sen^2x(1 - Sen^2x)] - b(1 - 3Sen^2xCos^2x)$$

$$P = a[2 - 2Sen^2xCos^2x - Sen^2xCos^2x] - b(1 - 3Sen^2xCos^2x)$$

$$P = a(2 - 3Sen^2xCos^2x) - b(1 - 3Sen^2xCos^2x)$$

$$P = 2a - 3aSen^2xCos^2x - b + 3bSen^2xCos^2x$$

$$P = (2a - b) + \underbrace{(3b - 3a)}_{0}Sen^2xCos^2x \longrightarrow 3a = 3b$$

$$\therefore \frac{a}{b} = 1$$

CLAVE: E

4. Si: $\dfrac{Senx}{Senx+Cosx+1}=\dfrac{\sqrt{3}+1}{2}$ Calcule: $Secx+Tanx$

Resolución:

$$\frac{Senx}{Senx+Cosx+1}=\frac{\sqrt{3}+1}{2}$$

$$\frac{Senx+Cosx+1}{Senx}=\frac{2}{\sqrt{3}+1}$$

$$\frac{Senx}{Senx}+\frac{Cosx}{Senx}+\frac{1}{Senx}=\frac{2}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$1+Cotx+Cscx=\sqrt{3}-1$$

$$Cscx+Cotx=\sqrt{3}-2$$

$$Cscx-Cotx=\frac{1}{\sqrt{3}-2}\times\frac{\sqrt{3}+2}{\sqrt{3}+2}$$

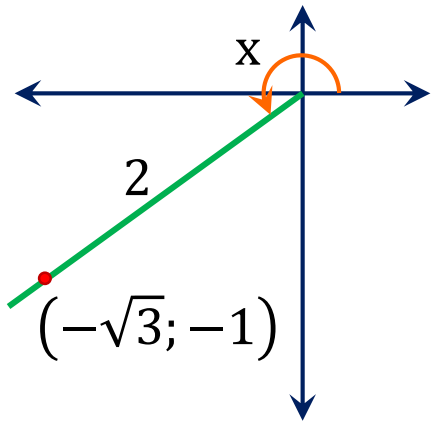
$$Cscx-Cotx=-2-\sqrt{3}$$

$$Cscx+Cotx=\sqrt{3}-2$$

$$Cscx-Cotx=-2-\sqrt{3}$$

$(+)$: $Cscx=-2$

$(-)$: $Cotx=\sqrt{3}$ $\longrightarrow$ $x\in IIIC$



$$E=Secx+Tanx$$

$$E=\frac{2}{-\sqrt{3}}+\frac{-1}{-\sqrt{3}}$$

$$E=-\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore \mathbf{E=-\frac{\sqrt{3}}{3}}$$

CLAVE: D

5. Si: $\text{Tanx} + \text{Cotx} = 5$; calcule: $\dfrac{\text{Tan}^3x}{1+\text{Tan}^2x} + \dfrac{\text{Cot}^3x}{1+\text{Cot}^2x}$

Resolución:

$$\text{Tanx} + \text{Cotx} = 5$$

$$\text{SecxCscx} = 5$$

$$\frac{1}{\text{SenxCosx}} = 5$$

$$\textbf{SenxCosx} = \frac{\mathbf{1}}{\mathbf{5}}$$

$$L = \frac{\text{Tan}^3x}{1+\text{Tan}^2x} + \frac{\text{Cot}^3x}{1+\text{Cot}^2x}$$

$$L = \frac{\text{Tan}^3x}{\text{Sec}^2x} + \frac{\text{Cot}^3x}{\text{Csc}^2x}$$

$$L = \frac{\dfrac{\text{Sen}^3x}{\text{Cos}^2x}}{\dfrac{1}{\text{Cos}^2x}} + \frac{\dfrac{\text{Cos}^3x}{\text{Sen}^2x}}{\dfrac{1}{\text{Sen}^2x}}$$

$$L = \frac{\text{Sen}^3x}{\text{Cosx}} + \frac{\text{Cos}^3x}{\text{Senx}}$$

$$L = \frac{\text{Sen}^4x + \text{Cos}^4x}{\text{SenxCosx}}$$

$$L = \frac{1 - 2\text{Sen}^2x\text{Cos}^2x}{\text{SenxCosx}}$$

$$L = \frac{1 - 2\left(\frac{1}{5}\right)^2}{\frac{1}{5}}$$

$$L = \frac{\frac{23}{25}}{\frac{1}{5}}$$

$$\therefore \mathbf{L = \frac{23}{5}}$$

CLAVE: B

5. Si: $\text{Tan}x + \text{Cot}x = 5$; calcule: $\dfrac{\text{Tan}^3x}{1+\text{Tan}^2x} + \dfrac{\text{Cot}^3x}{1+\text{Cot}^2x}$

Resolución:

$$\text{Tan}x + \text{Cot}x = 5$$

$$\text{Sec}x\text{Csc}x = 5$$

$$\text{Tan}^3x + 3(\text{Tan}x + \text{Cot}x).1 + \text{Cot}^3x = 125$$

$$\text{Tan}^3x + \text{Cot}^3x + 3(5) = 125$$

$$\text{Tan}^3x + \text{Cot}^3x = 110$$

$$L = \frac{\text{Tan}^3x}{1+\text{Tan}^2x} + \frac{\text{Cot}^3x}{1+\text{Cot}^2x}$$

$$L = \frac{\text{Tan}^3x(1+\text{Cot}^2x) + \text{Cot}^3x(1+\text{Tan}^2x)}{\text{Sec}^2x\text{Csc}^2x}$$

$$L = \frac{\text{Tan}^3x + \text{Tan}x + \text{Cot}^3x + \text{Cot}x}{(\text{Sec}x\text{Csc}x)^2}$$

$$L = \frac{\text{Tan}^3x + \text{Cot}^3x + 5}{5^2}$$

$$L = \frac{115}{25}$$

$$\therefore \mathbf{L = \frac{23}{5}}$$

CLAVE: B

6. Siendo: $\csc^2x + \sec^2x = 9$; calcule: $|\tan^3x(1 + \cot^8x) + \cot^3x(1 + \tan^8x)|$

Resolución:

$$J = |\tan^3x(1 + \cot^8x) + \cot^3x(1 + \tan^8x)|$$

$$J = |\tan^3x + \tan^3x\cot^8x + \cot^3x + \cot^3x\tan^8x|$$

$\cot^3x\cot^5x$ $\qquad$ $\tan^3x\tan^5x$

$$J = |\tan^3x + \cot^5x + \cot^3x + \tan^5x|$$

$$J = |(\tan x + \cot x)(\tan^4x + \cot^4x)|$$

± 3 $\qquad$ 47

$$J = |\pm 141|$$

$\therefore \mathbf{J = 141}$

$$\csc^2x + \sec^2x = 9$$

$$1 + \cot^2x + 1 + \tan^2x = 9$$

$$(\tan x + \cot x)^2 = 9$$

$$\tan x + \cot x = \pm 3$$

$$\tan^2x + \cot^2x = 7$$

AL^2: $\tan^4x + 2 + \cot^4x = 49$

$$\tan^4x + \cot^4x = 47$$

CLAVE: E

7. Elimine la variable angular x de las expresiones:

$$\text{Senx} + \text{Cosx} = \sqrt{a} \ldots (I) \qquad \text{Cotx} - \text{Tanx} = \sqrt{b} \ldots (II)$$

Resolución:

$$\text{Senx} + \text{Cosx} = \sqrt{a}$$

AL^2: $(\text{Senx} + \text{Cosx})^2 = (\sqrt{a})^2$

$$1 + 2\text{SenxCosx} = a$$

$$\text{SenxCosx} = \frac{a-1}{2}$$

$$\frac{2}{a-1} = \frac{1}{\text{SenxCosx}}$$

$$\text{SecxCscx} = \frac{2}{a-1}$$

$$\mathbf{Tanx + Cotx = \frac{2}{a-1}}$$

$$\text{Cotx} - \text{Tanx} = \sqrt{b} \qquad \text{Cotx} + \text{Tanx} = \frac{2}{a-1}$$

Por Legendre: $\mathbf{(S)^2 - (D)^2 = 4P}$

$$(\text{Cotx} + \text{Tanx})^2 - (\text{Cotx} - \text{Tanx})^2 = 4\underbrace{\text{CotxTanx}}_{1}$$

$$\left(\frac{2}{a-1}\right)^2 - (\sqrt{b})^2 = 4$$

$$\frac{4}{(a-1)^2} - b = 4$$

$$\therefore \mathbf{(b+4)(a-1)^2 = 4}$$

CLAVE: A

8. Siendo: $\text{Senx} - \text{Cosx} = 1/3$ calcule: $\dfrac{\frac{3}{8}(1+\text{Senx})(1-\text{Cosx})+\text{Cscx}}{\frac{3}{2}(1-\text{Senx})(1+\text{Cosx})+\text{Cscx}}$

Resolución:

$$A = \frac{\frac{3}{8}(1+\text{Senx})(1-\text{Cosx})+\text{Cscx}}{\frac{3}{2}(1-\text{Senx})(1+\text{Cosx})+\text{Cscx}}$$

$$A = \frac{\frac{3}{16}\times 2(1+\text{Senx})(1-\text{Cosx})+\text{Cscx}}{\frac{3}{4}\times 2(1-\text{Senx})(1+\text{Cosx})+\text{Cscx}}$$

$$A = \frac{\frac{3}{16}\times (1+\text{Senx}-\text{Cosx})^2+\text{Cscx}}{\frac{3}{4}\times (1-\text{Senx}+\text{Cosx})^2+\text{Cscx}}$$

$$A = \frac{\frac{3}{16}\times \left(1+\frac{1}{3}\right)^2+\text{Cscx}}{\frac{3}{4}\times \left(1-\frac{1}{3}\right)^2+\text{Cscx}}$$

$$A = \frac{\frac{3}{16}\left(\frac{16}{9}\right)+\text{Cscx}}{\frac{3}{4}\left(\frac{4}{9}\right)+\text{Cscx}}$$

$$A = \frac{\frac{1}{3}+\text{Cscx}}{\frac{1}{3}+\text{Cscx}}$$

$\therefore \mathbf{A = 1}$

CLAVE: C

9. Siendo: $\sqrt{\sqrt{5}+1}\text{Secx}-\sqrt{\sqrt{5}-1}\text{Tanx}=\sqrt{2}$ calcule: SecxTanx

Resolución:

$$\sqrt{\sqrt{5}+1}\text{Secx}-\sqrt{\sqrt{5}-1}\text{Tanx}=\sqrt{2}$$

$$\sqrt{\sqrt{5}+1}\times\frac{1}{\text{Cosx}}-\sqrt{\sqrt{5}-1}\times\frac{\text{Senx}}{\text{Cosx}}=\sqrt{2}$$

$$\sqrt{\sqrt{5}+1}-\sqrt{\sqrt{5}-1}\text{Senx}=\sqrt{2}\text{Cosx}$$

$$\underbrace{\sqrt{\sqrt{5}-1}}_{a}\text{Senx}+\underbrace{\sqrt{2}}_{b}\text{Cosx}=\underbrace{\sqrt{\sqrt{5}+1}}_{\sqrt{a^2+b^2}}$$

$$\text{Senx}=\frac{\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}+1}}\qquad \text{Cosx}=\frac{\sqrt{2}}{\sqrt{\sqrt{5}+1}}\qquad \text{Tanx}=\frac{\sqrt{\sqrt{5}-1}}{\sqrt{2}}$$

$$\text{E}=\text{SecxTanx}$$

$$\text{E}=\left(\frac{\sqrt{\sqrt{5}+1}}{\sqrt{2}}\right)\left(\frac{\sqrt{\sqrt{5}-1}}{\sqrt{2}}\right)$$

$$\text{E}=\left(\frac{\left(\sqrt{(\sqrt{5}+1)(\sqrt{5}-1)}\right)}{2}\right)$$

$$\text{E}=\left(\frac{\sqrt{4}}{2}\right)$$

$\therefore \mathbf{E=1}$

CLAVE: A

REPASO

10. Siendo: $\mathrm{Sec}^7x + \mathrm{Sec}^6x + \mathrm{Sec}^5x + \mathrm{Sec}^3x + \mathrm{Sec}^2x + \mathrm{Sec}x = 28\mathrm{Sec}^4x$ .Calcule k en la igualdad: $\mathrm{Tan}^4x - k\mathrm{Tan}^2x - k = 0$

CLAVE: C

PITAGORAS ACADEMIA

IDENTIDADES PARA EL ÁNGULO COMPUESTO

1. SUMA DE ÁNGULOS:

- $\text{Sen}(x + y) = \text{Senx}.\text{Cosy} + \text{Seny}.\text{Cosx}$
- $\text{Cos}(x + y) = \text{Cosx}.\text{Cosy} - \text{Senx}.\text{Seny}$
- $\text{Tan}(x + y) = \dfrac{\text{Tanx} + \text{Tany}}{1 - \text{Tanx}.\text{Tany}}$

2. DIFERENCIA DE ÁNGULOS:

- $\text{Sen}(x - y) = \text{Senx}.\text{Cosy} - \text{Seny}.\text{Cosx}$
- $\text{Cos}(x - y) = \text{Cosx}.\text{Cosy} + \text{Senx}.\text{Seny}$
- $\text{Tan}(x - y) = \dfrac{\text{Tanx} - \text{Tany}}{1 + \text{Tanx}.\text{Tany}}$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

3. IDENTIDADES AUXILIARES:

a) $\text{Sen}(x+y)\text{Sen}(x-y) = \text{Sen}^2x - \text{Sen}^2y$

b) $\text{Cos}(x+y)\text{Cos}(x-y) = \text{Cos}^2x - \text{Sen}^2y$

c) $\text{Tan}(x+y)\text{Tan}(x-y) = \dfrac{\text{Tan}^2x - \text{Tan}^2y}{1 - \text{Tan}^2x\text{Tan}^2y}$

d) $\text{Tan}x \pm \text{Tan}y = \dfrac{\text{Sen}(x \pm y)}{\text{Cos}x\text{Cos}y}$

e) $\text{Cot}x \pm \text{Cot}y = \dfrac{\text{Sen}(y \pm x)}{\text{Sen}y\text{Sen}x}$

f) $\text{Tan}x \pm \text{Tan}y \pm \text{Tan}(x \pm y).\text{Tan}x.\text{Tan}y = \text{Tan}(x \pm y)$

$\text{Tan}x \pm \text{Tan}y \pm N\text{Tan}x.\text{Tan}y = N$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

4. PROPIEDADES:

I) $aSenx \pm bCosx = \sqrt{a^2 + b^2}Sen(x \pm \varphi), a \text{ y } b \in \mathbb{R}^+$

$$\text{Donde: } Tan\varphi = \frac{b}{a}$$

II) Sea: $E = aSenx + bCosx, x \in \mathbb{R}$

$$\underbrace{-\sqrt{a^2 + b^2}}_{\text{mín}} \leq aSenx + bCosx \leq \underbrace{\sqrt{a^2 + b^2}}_{\text{máx}}$$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

5. PROPIEDADES PARA 3 ÁNGULOS:

- Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$Tanx + Tany + Tanz = Tanx.Tany.Tanz$$

$$Cotx.Coty + Cotx.Cotz + Coty.Cotz = 1$$

- Si: $x + y + z = (2n + 1)\frac{\pi}{2}; n \in \mathbb{Z}$

$$Cotx + Coty + Cotz = Cotx.Coty.Cotz$$

$$Tanx.Tany + Tanx.Tanz + Tany.Tanz = 1$$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

11. Si: $\text{Sen}\alpha = 2\text{Sen}\beta$ y $\text{Cos}\beta = 3\text{Cos}\alpha$ calcule el valor de $\text{Cos}(\alpha - \beta)$

Resolución:

$$\text{Cos}(\alpha - \beta) = \text{Cos}\alpha\underbrace{\text{Cos}\beta}_{3\text{Cos}\alpha} + \text{Sen}\alpha\underbrace{\text{Sen}\beta}_{\frac{1}{2}\text{Sen}\alpha}$$

$$\text{Cos}(\alpha - \beta) = 3\text{Cos}^2\alpha + \frac{1}{2}\text{Sen}^2\alpha$$

$$\text{Cos}(\alpha - \beta) = 3(1 - \text{Sen}^2\alpha) + \frac{1}{2}\text{Sen}^2\alpha$$

$$\text{Cos}(\alpha - \beta) = 3\left(1 - \frac{32}{35}\right) + \frac{1}{2}\left(\frac{32}{35}\right)$$

$$\text{Cos}(\alpha - \beta) = 3\left(\frac{3}{35}\right) + \left(\frac{16}{35}\right)$$

$$\therefore \mathbf{Cos(\alpha - \beta) = \frac{5}{7}}$$

$$\text{Sen}^2\beta + \text{Cos}^2\beta = 1$$

$$\left(\frac{1}{2}\text{Sen}\alpha\right)^2 + (3\text{Cos}\alpha)^2 = 1$$

$$\frac{1}{4}\text{Sen}^2\alpha + 9(1 - \text{Sen}^2\alpha) = 1$$

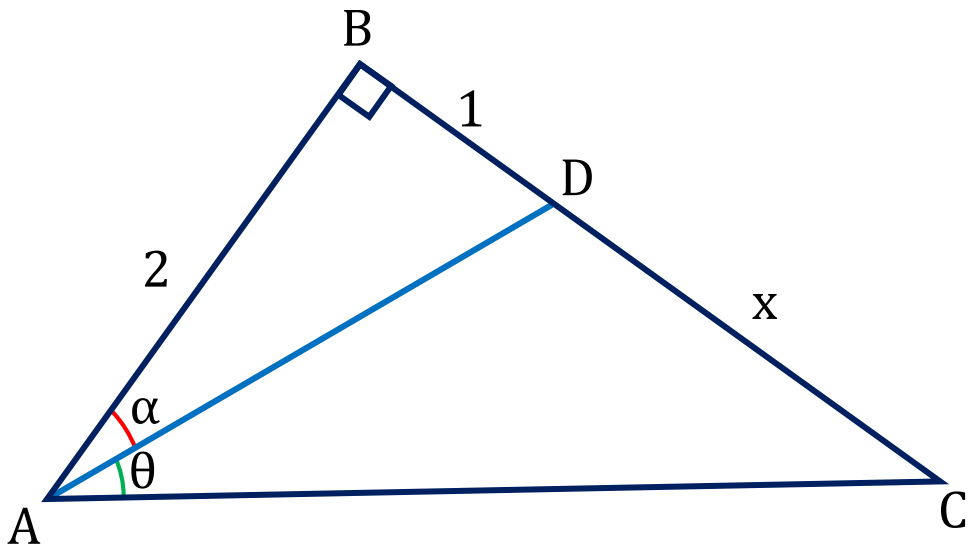
$$9 + \frac{1}{4}\text{Sen}^2\alpha - 9\text{Sen}^2\alpha = 1$$

$$\frac{35}{4}\text{Sen}^2\alpha = 8$$

$$\text{Sen}^2\alpha = \frac{32}{35}$$

CLAVE: D

12. En el gráfico se cumple que BD=1 y AB=2. Además $\text{Tan}(\theta-\alpha)=\frac{1}{5}$ Calcule CD.



Del gráfico: $\text{Tan}\,\alpha=\frac{1}{2}$

Resolución:

$$\text{Tan}(\theta-\alpha)=\frac{1}{5}$$

$$\frac{\text{Tan}\theta-\text{Tan}\,\alpha}{1+\text{Tan}\,\theta\,\text{Tan}\,\alpha}=\frac{1}{5}$$

$$\frac{\text{Tan}\theta-\frac{1}{2}}{1+\text{Tan}\,\theta\left(\frac{1}{2}\right)}=\frac{1}{5}$$

$$\frac{\frac{2\text{Tan}\theta-1}{2}}{\frac{2+\text{Tan}\theta}{2}}=\frac{1}{5}$$

$$10\text{Tan}\theta-5=2+\text{Tan}\theta$$

$$\text{Tan}\theta=\frac{7}{9}$$

$$\text{Tan}(\theta+\alpha)=\frac{x+1}{2}$$

$$\frac{\text{Tan}\theta+\text{Tan}\,\alpha}{1-\text{Tan}\,\theta\,\text{Tan}\,\alpha}=\frac{x+1}{2}$$

$$\frac{\frac{7}{9}+\frac{1}{2}}{1-\frac{7}{9}\times\frac{1}{2}}=\frac{x+1}{2}$$

$$\frac{\frac{14+9}{18}}{\frac{18-7}{18}}=\frac{x+1}{2}$$

$$\frac{23}{11}=\frac{x+1}{2}$$

$$\therefore x=\frac{35}{11}$$

CLAVE: C

13. Siendo: $aSenx + bCosx = c$ $aSeny + bCosy = d$ Eliminar x e y, si: $x - y = \theta$

Resolución:

$aSenx + bCosx = c$ ... (i)

$aSeny + bCosy = d$ ... (ii)

AL^2: $a^2Sen^2y + 2abSenyCosy + b^2Cos^2y = d^2$

$x - y = \theta$

$x = y + \theta$ ... (iii)

Reemplazando (iii)en (i):

$aSen(y + \theta) + bCos(y + \theta) = c$

$aSenyCos\theta + aSen\theta Cosy + bCosyCos\theta - bSenySen\theta = c$

$Cos\theta\underbrace{(aSeny + bCosy)}_{d} + Sen\theta(aCosy - bSeny) = c$

$$aCosy - bSeny = \frac{c - dCos\theta}{Sen\theta}$$

$$aCosy - bSeny = \frac{c - dCos\theta}{Sen\theta}$$

AL^2:

$$a^2Cos^2y - 2abSenyCosy + b^2Sen^2y = \left(\frac{c - dCos\theta}{Sen\theta}\right)^2$$

$$a^2Sen^2y + 2abSenyCosy + b^2Cos^2y = d^2$$

(+): $$a^2 + b^2 = d^2 + \left(\frac{c - dCos\theta}{Sen\theta}\right)^2$$

$$(a^2 + b^2)Sen^2\theta = d^2Sen^2\theta + c^2 - 2cdCos\theta + d^2Cos^2\theta$$

$$\therefore \mathbf{(a^2 + b^2)Sen^2\theta = d^2 + c^2 - 2cdCos\theta}$$

CLAVE: A

14. Calcule el mayor valor de x ($x < 270°$), correspondiente al máximo valor de la expresión: $W = Sen2x + \sqrt{3}Cos2x$

Resolución:

$$W = 1Sen2x + \sqrt{3}Cos2x$$

$$\underbrace{W}_{\text{Máx}} = \sqrt{1^2 + (\sqrt{3})^2}\,\underbrace{Sen(2x + 60°)}_{\text{Máx}}$$

$$Sen(\underbrace{2x + 60°}_{(4n+1)90°}) = 1$$

$$2x + 60° = 360°n + 90°$$

$$2x = 360°n + 30°$$

$$x = 180°n + 15°$$

$$x < 270°$$

$$180°n + 15° < 270°$$

$$n = 1 \longrightarrow 195° < 270°$$

$$x = 180°n + 15°$$

$$\therefore \mathbf{x = 195°}$$

CLAVE: B

15. Determine la variación de: $C = \text{Sen}\left(\frac{\pi}{x^2+4}\right) + \sqrt{3}\text{Cos}\left(\frac{\pi}{x^2+4}\right)$

Resolución:

$$C = 1\text{Sen}\underbrace{\left(\frac{\pi}{x^2+4}\right)}_{\theta} + \sqrt{3}\text{Cos}\underbrace{\left(\frac{\pi}{x^2+4}\right)}_{\theta}$$

$$C = 2\text{Sen}\left(\theta + \frac{\pi}{3}\right)$$

$$\frac{\sqrt{3}}{2} < \text{Sen}\left(\theta + \frac{\pi}{3}\right) \leq 1$$

$$\sqrt{3} < 2\text{Sen}\left(\theta + \frac{\pi}{3}\right) \leq 2$$

$$\sqrt{3} < C \leq 2$$

$$\therefore C \in \left]\sqrt{3}; 2\right]$$

$$\theta = \frac{\pi}{x^2+4}$$

$$x^2 \geq 0$$

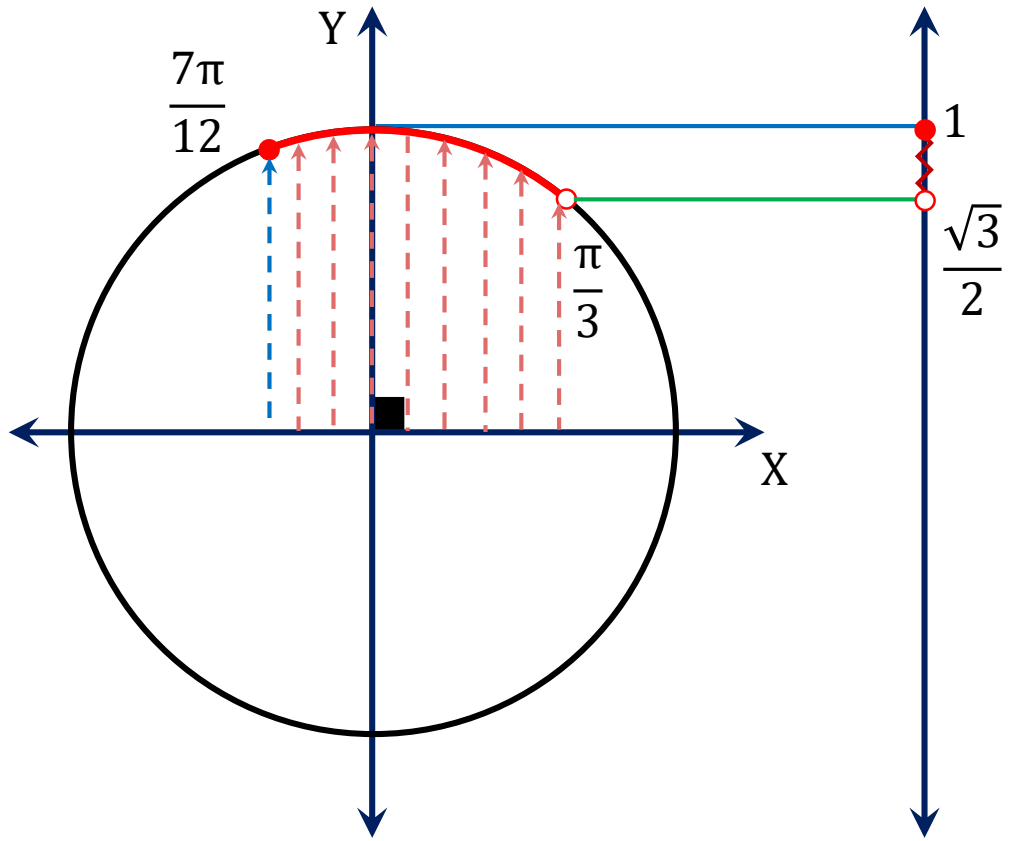
$$x^2 + 4 \geq 4$$

$$0 < \frac{1}{x^2+4} \leq \frac{1}{4}$$

$$0 < \frac{\pi}{x^2+4} \leq \frac{\pi}{4}$$

$$0 < \theta \leq \frac{\pi}{4}$$

$$\frac{\pi}{3} < \theta + \frac{\pi}{3} \leq \frac{7\pi}{12}$$



CLAVE: A

16. Si: $(m+n)Tan4\theta = mTan7\theta + nTan\theta$ Exprese: $Cos7\theta Sec\theta$ en términos de m y n.

Resolución:

$$(m+n)Tan4\theta = mTan7\theta + nTan\theta$$

$$mTan4\theta + nTan4\theta = mTan7\theta + nTan\theta$$

$$n(Tan4\theta - Tan\theta) = m(Tan7\theta - Tan4\theta)$$

$$n\left(\frac{\cancel{Sen3\theta}}{\cancel{Cos4\theta}Cos\theta}\right) = m\left(\frac{\cancel{Sen3\theta}}{Cos7\theta\cancel{Cos4\theta}}\right)$$

$$n\left(\frac{1}{Cos\theta}\right) = m\left(\frac{1}{Cos7\theta}\right)$$

$$\therefore \mathbf{Cos7\theta Sec\theta = \frac{m}{n}}$$

CLAVE: A

17. Calcule el valor de: $\frac{Sen26°}{Cos43°Cos21°}+\frac{Sen21°}{Cos43°Cos26°}+\frac{Sen43°}{Cos26°Cos21°}$

Resolución:

$$E=\frac{Sen26°}{Cos43°Cos21°}+\frac{Sen21°}{Cos43°Cos26°}+\frac{Sen43°}{Cos26°Cos21°}$$

$$E=\frac{Cos64°}{Cos43°Cos21°}+\frac{Cos69°}{Cos43°Cos26°}+\frac{Cos47°}{Cos26°Cos21°}$$

$$E=\frac{Cos(43°+21°)}{Cos43°Cos21°}+\frac{Cos(43°+26°)}{Cos43°Cos26°}+\frac{Cos(26°+21°)}{Cos26°Cos21°}$$

$$E=\frac{Cos43°Cos21°-Sen43°Sen21°}{Cos43°Cos21°}+\frac{Cos43°Cos26°-Sen43°Sen26°}{Cos43°Cos26°}+\frac{Cos26°Cos21°-Sen26°Sen21°}{Cos26°Cos21°}$$

$$E=1-Tan43°Tan21°+1-Tan43°Tan26°+1-Tan26°Tan21°$$

$$E=3-\underbrace{(Tan43°Tan21°+Tan43°Tan26°+Tan26°Tan21°)}_{1}$$

$43°+26°+21°=90°$

$\therefore E=2$

CLAVE: B

18. En un triángulo ABC, halle el equivalente de la expresión: $\left[\frac{Sen^2A - Sen^2C}{Sen(A-C)CosACosC} + TanB\right] CotBCotC$

Resolución:

$$J = \left[\frac{Sen(A+C)\cancel{Sen(A-C)}}{\cancel{Sen(A-C)}CosACosC} + TanB\right] CotBCotC$$

$$J = \left[\frac{Sen(A+C)}{CosACosC} + TanB\right] CotBCotC$$

$$J = [TanA + TanC + TanB]CotBCotC$$

$$J = [TanATanCTanB]CotBCotC$$

$$\therefore \mathbf{J = TanA}$$

$$A + B + C = 180°$$

$$TanA + TanB + TanC = TanA.TanB.TanC$$

CLAVE: A

19. Si: $\alpha+\beta+\theta=\frac{\pi}{2}$ y además son agudos. Halle el menor valor que toma R.

$$R=\frac{Tan\alpha+2Tan\beta+3Tan\theta}{Cot\alpha}+\frac{Tan\beta+2Tan\theta+3Tan\alpha}{Cot\beta}+\frac{Tan\theta+2Tan\alpha+3Tan\beta}{Cot\theta}$$

Resolución:

$$\alpha+\beta+\theta=\frac{\pi}{2} \longrightarrow \mathbf{Tan\alpha.Tan\beta+Tan\alpha.Tan\theta+Tan\theta.Tan\beta=1}$$

$$R=\frac{Tan\alpha+2Tan\beta+3Tan\theta}{\frac{1}{Tan\alpha}}+\frac{Tan\beta+2Tan\theta+3Tan\alpha}{\frac{1}{Tan\beta}}+\frac{Tan\theta+2Tan\alpha+3Tan\beta}{\frac{1}{Tan\theta}}$$

$$R=Tan^2\alpha+2Tan\alpha Tan\beta+3Tan\theta Tan\alpha+Tan^2\beta+2Tan\theta Tan\beta+3Tan\alpha Tan\beta+Tan^2\theta+2Tan\alpha Tan\theta+3Tan\beta Tan\theta$$

$$R=Tan^2\alpha+Tan^2\beta+Tan^2\theta+5\underbrace{(Tan\alpha Tan\beta+Tan\theta Tan\beta+Tan\alpha Tan\theta)}_{1}$$

$$R=Tan^2\alpha+Tan^2\beta+Tan^2\theta+5$$

19. Si: $\alpha+\beta+\theta=\frac{\pi}{2}$ y además son agudos. Halle el menor valor que toma R.

$$\underbrace{R}_{\text{Mín}}=\underbrace{\text{Tan}^2\alpha+\text{Tan}^2\beta+\text{Tan}^2\theta}_{\text{Mín}}+5$$

$$R=1+5$$

$$\therefore R_{Mín}=6$$

MA ≥ MG

$$\frac{\text{Tan}^2\alpha+\text{Tan}^2\beta}{2}\geq\sqrt{\text{Tan}^2\alpha\text{Tan}^2\beta}\rightarrow\frac{\text{Tan}^2\alpha+\text{Tan}^2\beta}{2}\geq\text{Tan}\alpha\text{Tan}\beta$$

$$\frac{\text{Tan}^2\alpha+\text{Tan}^2\theta}{2}\geq\sqrt{\text{Tan}^2\alpha\text{Tan}^2\theta}\rightarrow\frac{\text{Tan}^2\alpha+\text{Tan}^2\theta}{2}\geq\text{Tan}\alpha\text{Tan}\theta$$

$$\frac{\text{Tan}^2\theta+\text{Tan}^2\beta}{2}\geq\sqrt{\text{Tan}^2\theta\text{Tan}^2\beta}\rightarrow\frac{\text{Tan}^2\theta+\text{Tan}^2\beta}{2}\geq\text{Tan}\theta\text{Tan}\beta$$

(+)

$$\text{Tan}^2\alpha+\text{Tan}^2\beta+\text{Tan}^2\theta\geq 1$$

CLAVE: D

20. Si: $\mathrm{TanA}+\mathrm{TanB}+\mathrm{TanC}=0$ Además: $\mathrm{Tan}(\mathrm{A}+\mathrm{B})=\mathrm{CotC}(1+\mathrm{TanA})$ Simplifique:

$$\mathrm{M}=\left[\frac{\mathrm{Tan}^3\mathrm{A}+\mathrm{Tan}^3\mathrm{B}+\mathrm{Tan}^3\mathrm{C}}{\mathrm{Tan}^2\mathrm{ATanB}-1-\mathrm{TanA}}\right]\times\left[\frac{\mathrm{TanB}}{\mathrm{TanATanC}}+\frac{\mathrm{TanC}}{\mathrm{TanATanB}}+\frac{\mathrm{TanA}}{\mathrm{TanBTanC}}\right]$$

Resolución:

$$\mathrm{Tan}(\mathrm{A}+\mathrm{B})=\mathrm{CotC}(1+\mathrm{TanA})$$

$$\frac{\mathrm{TanA}+\mathrm{TanB}}{1-\mathrm{TanATanB}}=\frac{1}{\mathrm{TanC}}\times(1+\mathrm{TanA})$$

$$\mathrm{TanC}(\mathrm{TanA}+\mathrm{TanB})=(1+\mathrm{TanA})(1-\mathrm{TanATanB})$$

$$\mathrm{TanATanC}+\mathrm{TanBTanC}=1-\mathrm{TanATanB}+\mathrm{TanA}-\mathrm{Tan}^2\mathrm{ATanB}$$

$$\mathrm{Tan}^2\mathrm{ATanB}-1-\mathrm{TanA}=-(\mathrm{TanATanB}+\mathrm{TanATanC}+\mathrm{TanBTanC})$$

Resolución:

$$M = \left[\frac{Tan^3A + Tan^3B + Tan^3C}{Tan^2ATanB - 1 - TanA}\right] \times \left[\frac{TanB}{TanATanC} + \frac{TanC}{TanATanB} + \frac{TanA}{TanBTanC}\right]$$

$$M = \left[\frac{Tan^3A + Tan^3B + Tan^3C}{Tan^2ATanB - 1 - TanA}\right] \times \left[\frac{Tan^2A + Tan^2B + Tan^2C}{TanATanBTanC}\right]$$

$$TanA + TanB + TanC = 0 \quad \rightarrow Tan^3A + Tan^3B + Tan^3C = 3TanATanBTanC$$

$$\rightarrow Tan^2A + Tan^2B + Tan^2C = -2(TanATanB + TanATanC + TanBTanC)$$

$$M = \left[\frac{3TanATanBTanC}{-(TanATanB + TanATanC + TanBTanC)}\right] \times \left[\frac{-2(TanATanB + TanATanC + TanBTanC}{TanATanBTanC}\right]$$

$\therefore \mathbf{M = 6}$

CLAVE: E

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS